

The particle emitter size dependence on energy in e^+e^- annihilations into hadrons

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Abstract

The nearly energy independent hadron emitter dimension r , measured in e^+e^- annihilation in the energy range 10 to 91 GeV via the Bose-Einstein correlation of two identical charged pions, is shown to be well accounted for by choosing the hadron jets as independent pion sources. To this end the known normalised factorial cumulant moments dependence on particle sources is adapted to the Bose-Einstein correlation formalism to yield a relation between r and these sources. This approach is also able to account for the measured r values obtained for the Z^0 decays into two and three hadron jets. Finally the estimated r value of the hadronic $\Upsilon(9.46)$ decay via three gluons is expected to be higher by about 6 to 11% over that predicted for its one photon hadronic decay mode.

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1 Introduction

The two-particle intensity interferometry has been proposed by Hanbury-Brown and Twiss [1] to correlate radio waves arriving from outer space for the purpose of measuring the angular diameter of astronomical objects. This method, which here is referred to as Bose-Einstein correlations (BEC), has been further extended by Goldhaber et al., [2] to the system of identical hadron pairs produced in two-particle collisions. Taking the hadron emitter to be a sphere of radius r with a Gaussian distribution, the correlation function $C(p_1, p_2)$ of pairs of identical bosons with momenta p_1 and p_2 can be parametrised by

$$C(Q) = 1 + \lambda e^{-r^2 Q^2}, \quad (1)$$

where $Q^2 = -(p_1 - p_2)^2$. The λ parameter, usually referred to as the chaoticity parameter, is a measure of the incoherence level of the hadrons source. However, the estimated λ values obtained from BEC analyses are also affected by the purity level of the identical hadron pairs sample taken from the experimental data. In recent years the BEC interferometry of pairs of charged pions is extensively used in order to investigate the underlying dynamical processes occurring in heavy ion collisions at ultra high energies [3]. In parallel, the BEC analysis was and is also applied to e^+e^- annihilations at different centre-of-mass energies \sqrt{s} , and as a function of the outgoing hadron mass, in order to estimate the emitter dimension [4]. Lately a special attention has been given to the role, if any, of these BEC in the reaction $e^+e^- \rightarrow W^+W^- \rightarrow$ hadrons in order to estimate their possible effect on the W mass determination [5].

An early compilation of the $r_{\pi^\pm\pi^\pm}$ values measured in heavy ions collisions [6] is seen in Fig. 1 to be rather well described by the straight line representing the relation $r = 1.2A^{1/3}$ fm, the known nucleus radius dependence on the atomic number A . This increase is attributed to the rise of the number of hadron sources, such as the nucleon-nucleon interaction, which occur as the atomic number of the colliding ions increases [3]. At the same time the BEC deduced r values, extracted from e^+e^- annihilations leading mainly to pions, seem to be rather independent of the centre of mass energy of the colliding electrons. This is illustrated in Fig. 2 where the small deviations from a constant r value are attributed to the variation in the experimental procedures adopted in each of the BEC analyses [4, 7]. Additional information concerning the properties of the BEC extracted r dimension is coming from the hadronic Z^0 decays where it is found that the r value increases with number of hadron jets and charged pion multiplicity [8].

Here it is shown that the observed energy independent behaviour of r in $e^+e^- \rightarrow$ hadrons annihilations is consistent with the assumption that only hadrons emerging from the same final state hadron jet can be correlated. In addition, this approach leads also to an r increase of some 10% of the three-jet hadronic Z^0 decays over that expected for two-jets decay events. Finally the hadron jet source approach is applied to the hadronic $\Upsilon(9.46)$ decays via the three gluons and one photon processes.

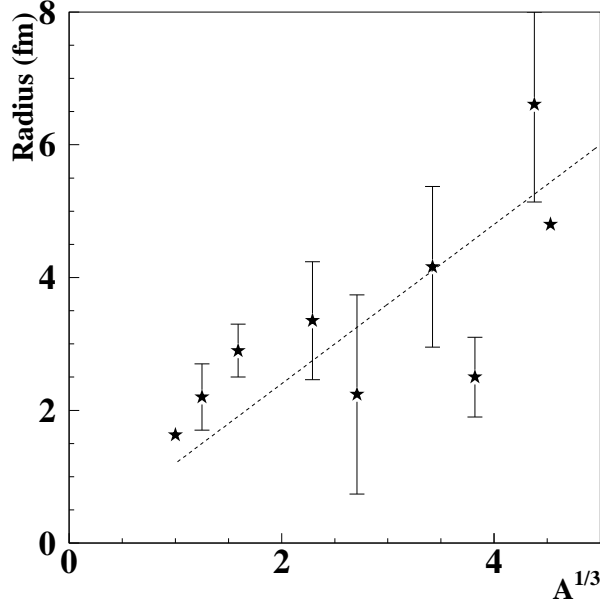


Figure 1: A compilation of $r_{\pi^{\pm}\pi^{\pm}}$ extracted from BEC analyses of identical charged pion-pairs produced in heavy ion collisions shown as a function of $A^{1/3}$ where A is the atomic number of the projectile. The values are taken from Ref. [6] and whenever more than one value was given for the same projectile the plotted value is their average. The line represents the relation $r = 1.2A^{1/3}$ fm.

2 The hadron source approach in cumulants and BEC

A direct way to extract the genuine dynamical correlations of two and more hadrons, produced in high energy reactions, is offered by the so called bin-averaged normalised factorial cumulant moments K_q , here denoted simply as cumulants, where q is the size of the hadron group and is equal e.g. to 2 for a pion-pair system. This method, which was first proposed in [9] and described in details in [10], has been applied in the study of multi-particle dynamical fluctuations. In these investigations, also referred to as intermittency analyses, the dependence of the cumulants values on the average multiplicity has been proposed to be the consequence of a mixing of several well defined hadron emission sources [11–15].

In this emission source approach, correlations exist only between particles emitted from the same source so that the correlation strength is reduced whenever particles are grouped from more than one source. For a quantitative evaluation of this reduction we follow here closely Ref. [15] in its treatment of the one dimensional (rapidity) cumulants K_q^{av} , which are averaged over M bins with the condition that $M \geq 10$. If one denotes the average value of a cumulant determined from one hadron source as $K_q^{av(1)}$ then the cumulant $K_q^{av(k)}$, calculated for all possible q-groups from a state k of more than one source, will be diluted by a factor $D_q^{(k)}$ so that

$$K_q^{av(k)} = D_q^{(k)} K_q^{av(1)}, \quad (2)$$

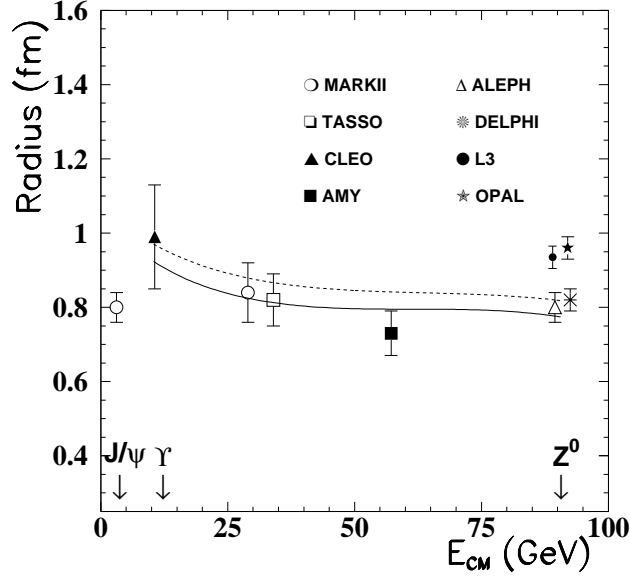


Figure 2: Values of $r_{\pi^+\pi^-}$ obtained from Bose-Einstein correlation analyses of hadrons produced in e^+e^- annihilations [4, 7]) plotted against \sqrt{s} . The shown band is the $r_{\pi^+\pi^-}$ behaviour expected for a constant λ value from the hadron source approach, normalised at 91 GeV to the r value obtained by the ALEPH collaboration. The continuous and dashed lines correspond respectively to the x_{max} values of 0.99 and 0.95 used in calculating $\langle D_{qqg} \rangle$ via Eq. (12).

where $D_q^{(k)}$ is equal to

$$D_q^{(k)} = \frac{P_q^G}{(P_q^G + P_q^{NG})} . \quad (3)$$

Here P_q^G denotes the number of q-particle groups, e.g. pairs of pions, emerging from the same source while P_q^{NG} stands for the number of all possible combinations of q-particle groups which emerge from at least two sources. In Ref. [15] it has further been shown that in the case of S identical sources, each having the same charged multiplicity n , one has for $q = 2$ that

$$D_{q=2}^{(k)} = \frac{\binom{n}{2} S}{\binom{n}{2} S + n^2 \binom{S}{2}} \xrightarrow{n \gg 1} \frac{1}{S} . \quad (4)$$

This expression for $q = 2$ can be generalised for $q \geq 1$ to yield $D_q \xrightarrow{n \gg q} 1/S^{q-1}$. For the present work it is important to realise that even in the case where the number of sources remains the same, but the condition of equal multiplicity is lifted, the dilution factor D_q may change when the over-all multiplicity and/or its division between the sources is modified.

In the case of two particle correlations, which obviously are genuine, one is able to relate [16] the normalised cumulant K_2 and the BEC correlation function $C(p_1, p_2)$, namely:

$$C(p_1, p_2) - 1 = K_2(p_1, p_2) . \quad (5)$$

A comparison between this expression and Eq. (1) affords the possibility to utilise the dependence of the averaged normalised cumulant on the hadron sources and their charged multiplicity configuration to the BEC correlation term $\lambda e^{-r^2 Q^2}$ average over the Q variable. Specifically, let us consider the ratio of the normalised average cumulants $K_2^{av(m)}/K_2^{av(k)}$, where the superscripts m and k denote two different hadron sources configurations. Thus one has

$$K_2^{av(m)}/K_2^{av(k)} = \langle C^{(m)}(Q) - 1 \rangle / \langle C^{(k)}(Q) - 1 \rangle = \int_0^\infty \lambda_m e^{-r_m^2 Q^2} dQ / \int_0^\infty \lambda_k e^{-r_k^2 Q^2} dQ = \frac{\lambda_m r_k}{\lambda_k r_m}.$$

From Eq. (2) follows that $K_2^{av(m)}/K_2^{av(k)} = D_2^{(m)}/D_2^{(k)}$ so that one finally obtains a simple relation between the hadron emitter radii and the dilution factors, namely

$$r_m = \frac{\lambda_m}{\lambda_k} \frac{D_2^{(k)}}{D_2^{(m)}} r_k. \quad (6)$$

If we set the final state k to be that emerging from a single source and assign to it the value $k = 1$, then $D_2^{(1)}$ serves as a yardstick against which dilution factors of other final states are measured. Thus for identical sources having the same λ and average charged multiplicity, Eq. (6) reduces with the help of relation (4) to $r_m = S r_1$ where r_1 is the emitter radius of a single hadron source. Hence r increases linearly with S , the number of hadron sources. Although it seems a priori naturally to choose for the heavy ions reactions the nucleon-nucleon collision as the basic single hadron source, the actual description of nuclei reaction is much more complicated mainly due to the secondary interactions within the nucleus.

3 $r_{\pi^\pm \pi^\pm}$ versus \sqrt{s} in e^+e^- annihilation

In the following we propose to study the dependence of $r_{\pi^\pm \pi^\pm}$ on \sqrt{s} , as measured in the $e^+e^- \rightarrow$ hadrons annihilation, in terms of the hadron sources given by the quark and gluon jets. Experimentally, for the grouping of the outgoing hadrons into jets there exist several methods which differ somewhat in their results. Among the leading ones is the known Durham algorithm [17], which has been applied to the hadronic Z^0 decay. The outcome of such a hadron jet analysis, as obtained by the OPAL collaboration [18], is shown in Fig. 3 as a function of the jet resolution parameter y_{cut} . As can be seen, already for values of $y_{cut} \geq 0.01$, the data is predominantly limited by three hadron jets. In the following we will assume that at $\sqrt{s} \leq 91$ GeV the $e^+e^- \rightarrow$ hadrons annihilation is governed by not more than the three-jet configuration.

In the first next-to-leading process $e^+e^- \rightarrow q\bar{q}g$, the differential cross section is given by (see e.g. Ref. [19]):

$$\frac{1}{\sigma} \frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} = \frac{2\alpha_s}{3\pi} W_{qqg}(x_1, x_2). \quad (7)$$

Here $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ where E_q and $E_{\bar{q}}$ are respectively the energies of the outgoing quark and anti-quark jets. At this point it is important to realise that the term

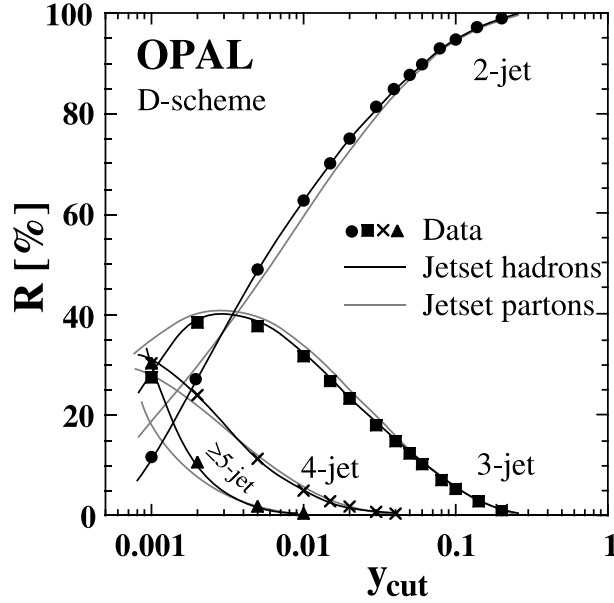


Figure 3: The relative measured rate R of the produced hadron jets in the Z^0 decay, taken from reference [18], as a function of the jet resolution parameter y_{cut} using the Durham algorithm. The data which is corrected for all the OPAL detector deficiencies, but not for hadronisation effects, is compared with two Monte Carlo models.

$W_{q\bar{q}g}(x_1, x_2)$ is equivalent to the probability of finding a gluon with the energy $E_g = \sqrt{s} - E_q - E_{\bar{q}}$ which here is utilised for the calculation of the dilution factors. The integration region associated to this differential cross section is defined by $0 \leq x_1, x_2 \leq 1$ and $x_1 + x_2 \geq 1$. Further to note is that the expression given by Eq. (7) diverges when either the gluon is collinear with one of the outgoing quarks or the gluon momentum approaches zero. Some methods to handle in the integration these singularities are discussed e.g. in references [19,20]. Here we deal with this singularity by introducing to the x_1 and x_2 variables an upper limit of $x_{max} = 0.99$. To estimate the sensitivity of our calculations to this upper limit cutoff choice we also represent our analysis results using $x_{max} = 0.95$.

For the average charged multiplicity of the quark and anti-quark jets, in the process $e^+e^- \rightarrow q\bar{q}g$, it was found that its parametrisation is best given in terms of an appropriate scale defined as [21]

$$Q_{qg} = E_q \sin\left(\frac{\theta_{qg}}{2}\right) \quad \text{and} \quad Q_{\bar{q}g} = E_{\bar{q}} \sin\left(\frac{\theta_{\bar{q}g}}{2}\right), \quad (8)$$

where E_q ($E_{\bar{q}}$) is the energy of the quark (anti-quark) jet and θ_{qg} ($\theta_{\bar{q}g}$) is the opening angle between the quark (anti-quark) jet and the gluon jet. The effective scale for the gluon is then

$$Q_g = \sqrt{Q_{qg}Q_{\bar{q}g}}. \quad (9)$$

For the dependence of the multiplicity on the scale Q_{jet} we use the parametrisation given

in Refs. [19, 21], namely:

$$\langle N_q(Q_{jet}) \rangle = a_0 + a_1 \ln Q_{jet} + a_2 (\ln Q_{jet})^2 \quad (10)$$

and

$$\langle N_g(Q_{jet}) \rangle = R_0 + R_1 \langle N_q(Q_{jet}) \rangle, \quad (11)$$

where $a_0 = 2.74 \pm 0.07$, $a_1 = 1.71 \pm 0.05$, $a_2 = -0.05 \pm 0.07$, $R_0 = -7.27 \pm 0.52$ and $R_1 = 2.27 \pm 0.07$. With the help of these two parametrisations one can calculate the 2-pion dilution factor $D_{qqg}(x_1, x_2)$, as defined in Eq. (3), for every set of x_1, x_2 and $x_g = 2 - x_1 - x_2$ values assuming for simplicity that each jet charged multiplicity is equally divided into positive and negative pions. Thus the average 2-pion dilution factor $\langle D_{qqg} \rangle$, at a given e^+e^- centre-of-mass energy, is equal to

$$\langle D_{qqg} \rangle = \int_0^{x_{max}} dx_2 \int_{1-x_2}^{x_{max}} D_{qqg}(x_1, x_2) W_{qqg}(x_1, x_2) dx_1 / \int_0^{x_{max}} dx_2 \int_{1-x_2}^{x_{max}} W_{qqg}(x_1, x_2) dx_1. \quad (12)$$

The average dilution factors $\langle D_{qqg} \rangle$ were thus calculated at several \sqrt{s} values in the range of 10 to 91 GeV using the two x_{max} values of 0.99 and 0.95. Our results for the expected $r_{\pi^\pm\pi^\pm}$ dependence on energy, using Eq. (6) and assuming a constant λ value, are represented in Fig. 2 by a band chosen to be normalised to the r value measured by ALEPH [22] at 91 GeV so as to cover also r measurements at lower energies. The lower and upper lines of the band, which correspond respectively to the x_{max} values of 0.99 and 0.95, are seen to be very similar in their shape. As can be seen, the expected dependence of r on the energy is weak and consistent with the experimental observations in the energy range from 10 GeV up to the r values measured at the Z^0 mass energy by the ALEPH and DELPHI collaborations. At the same time, in the framework of the hadron source approach, no relation can be achieved between the reported r values of L3 and OPAL at $\sqrt{s} = 91$ GeV and those measured at lower energies. Finally important to note is that whereas in the description of the jet multiplicity via Eqs. (10) and (11) the use of Q_{jet} scale is mandatory, [21], the average calculated dilution factor $\langle D_{qqg} \rangle$ differs, in the 10 to 91 GeV energy range, only by about 2% when the Q_{jet} scale is replaced by the jet energy E_{jet} .

3.1 The $r_{\pi^\pm\pi^\pm}$ value of the Z^0 decay into two and three jets

The dependence of the emitter size on the number of hadron jets in the Z^0 decays has been studied by the OPAL collaboration [8] which found that the r value of the 3-jet events has a value which is higher by about 10% than that obtained for the 2-jet events. From the same study OPAL also reported that r increases with the observed charged multiplicity while the λ parameter decreases. As mentioned before, the experimental separation between 2-jet and 3-jet events depends on the method chosen and its parameters values and therefore its reproduction with Eq. (7) is inaccessible. On the other hand, transition from a 2-jet to a 3-jet Z^0 decay configuration can be characterised, in the framework of the hadron source model, by an increase of N_g , the number of charged hadrons associated with the gluon jet. The expected ratio of

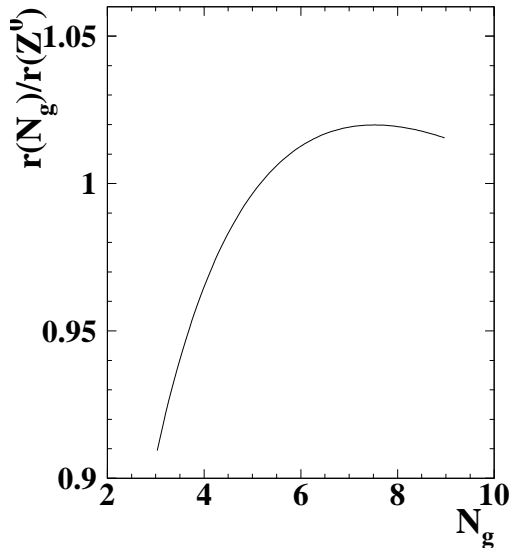


Figure 4: The expected ratio $r(N_g)/r(Z^0)$ of the hadronic Z^0 decay as a function of N_g , the gluon jet charged multiplicity, as calculated from the hadron source approach.

$r(N_g)/r(Z^0)$, as obtained from the model using $x_{max} = 0.99$, is shown in Fig. 4 as a function of N_g , where $r(Z^0)$ is the dimension value obtained from the BEC analysis of the total Z^0 final state hadrons. As seen, this ratio increases by about 10% from N_g of about three to values above seven where the ratio settles at a level of about ~ 1.02 . Similar behaviour of $r(N_g)/r(Z^0)$ is also seen when the value $x_{max} = 0.95$ is used in the calculations. Since $r(Z^0)$ lies between the r values of the 2 and 3-jet events, the model expectation is consistent with the experimental findings.

For the comparison of the OPAL results for the r dependence on multiplicity at the Z^0 decay [8], with the hadronic jet source approach one requires above all a parametrisation of the hadron multiplicity spectrum as a function of Q_{jet} in addition to its average value $\langle N(Q_{jet}) \rangle$. For the gluon jet this requirement can be achieved by using its hadron multiplicity distributions given in [23] for several Q_{jet} values. As for the quark jet, one can obtain the needed parametrisation by using the known average multiplicity versus Q_{jet} and by assuming that the shape of the multiplicity distribution is independent of the jet energy. With these parametrisations we find that for a constant λ the dimension r indeed increases with the true multiplicity. Nevertheless, it is not surprising that we are unable to reproduce the experimental findings in all their details and that for the following reasons. Firstly, the measured multiplicity given in [8] is the observed one, which depends on the experimental setup and analysis procedures, whereas the hadron jets parametrisations refer to the true multiplicity. Secondly, the measured chaoticity λ is found to decrease rather steeply with the multiplicity which may indicate an increase in the experimental data contamination as the number of outgoing particles increases. Thirdly, the present study has limited itself to the case where the decay configurations of more than three hadron jets can be neglected. While this assumption is supported by the OPAL analysis [18]

even at $\sqrt{s} = 91$ GeV, as long as the BEC analysis embraces the whole final state hadrons, it may not be anymore valid when only the higher multiplicity part of the hadronic Z^0 decay events is selected for the correlation study.

3.2 Quarkonium decay into hadrons

Quarkonia, like the $J/\psi(3.1)$ and the $\Upsilon(9.46)$, decay into hadrons predominantly via the three gluon ($3g$) mode, also referred to as the direct decay mode, which leads to three hadron jets. In addition, they also decay via the one photon, or vacuum polarisation, mode which is identical in its hadron jet configuration to that of the $e^+e^- \rightarrow \gamma/Z^* \rightarrow q\bar{q}g$ background underneath. The $J/\psi(3.1)$ mass is too low to be analysed here in terms of its hadron sources, mainly due to the uncertainty in the gluon and quark jet parametrisations below $\sim 1 - 2$ GeV. As for the $\Upsilon(9.46)$, its mass may be considered just heavy enough to render, according to Eq. (6), a reliable estimate of the ratio

$$\frac{r_{3g}}{r_{q\bar{q}g}} = \frac{\langle D_{q\bar{q}g} \rangle}{\langle D_{3g} \rangle} \frac{\lambda_{3g}}{\lambda_{q\bar{q}g}}. \quad (13)$$

To calculate the dilution factor of the three gluon-jet configuration we consider the differential $\Upsilon(9.46)$ direct decay rate given by [24]

$$\frac{1}{\Gamma_{3g}} \frac{d\Gamma}{dx_1 dx_2} = \frac{6}{\pi^2 - 9} W(x_1, x_2, x_3), \quad (14)$$

where $x_j = 2E_{g_j}/m_\Upsilon$. Here $W(x_1, x_2, x_3)$ is given by [24]

$$W(x_1, x_2, x_3) = \frac{1}{x_1^2 x_2^2 x_3^2} [x_1^2(1-x_1)^2 + x_2^2(1-x_2)^2 + x_3^2(1-x_3)^2] \quad (15)$$

with the condition that $x_1 + x_2 + x_3 = 2$. Whereas for the calculation of the average dilution factor $\langle D_{q\bar{q}g} \rangle$ one has a well defined Q_{jet} scale for the quarks and gluon jets as formulated in Eqs. (10) and (11), not so for the three gluon configuration. However, as mentioned before, the average dilution factors do change only by $\sim 2\%$ when the Q_{jet} scale is replaced by E_{jet} . This being the case we proceeded to calculate $r_{3g}/r_{q\bar{q}g}$ using the jet energy E_{jet} rather than the Q_{jet} scale while incorporating a 2% uncertainty into our final result. The evaluation of the average dilution factor $\langle D_{3g} \rangle$ is then carried out via Eq. (12) where $D_{q\bar{q}g}(x_1, x_2)$ and $W_{q\bar{q}g}(x_1, x_2)$ are replaced respectively by $D_{3g}(x_1, x_2)$ and $W_{3g}(x_1, x_2)$. From these calculations and those carried out to obtain $\langle D_{q\bar{q}g} \rangle$ at $\sqrt{s} = 9.46$ GeV, we find that the ratio $\langle D_{q\bar{q}g} \rangle / \langle D_{3g} \rangle$ is lying in the range of 1.06 to 1.11 corresponding to the integration cutoff values $x_{max} = 0.99$ and 0.95.

An early BEC analysis of the CLEO collaboration [25] yielded the values of $r(\Upsilon \rightarrow 3g) = 0.99 \pm 0.14$ fm and $r(\Upsilon \rightarrow q\bar{q}g) = 0.86 \pm 0.15$ fm so that their ratio is equal to 1.15 ± 0.26 . A more recent BEC measurements of $e^+e^- \rightarrow$ hadrons at the Υ mass energy, described in Ref. [26], quotes the values $r(\Upsilon \rightarrow 3g) = 0.73 \pm 0.10 \pm 0.04$ fm and $r(\Upsilon \rightarrow q\bar{q}g) = 0.83 \pm 0.22 \pm 0.05$ fm, from which follows that $r(\Upsilon \rightarrow 3g)/r(\Upsilon \rightarrow q\bar{q}g) = 0.88 \pm 0.28$. To compare these results

with the expectation of the hadron source approach one needs to know the true λ_{3g} and λ_{qqg} values. However as mentioned before, these are difficult to estimate due to the fact that they are sensitive to the purity level of the experimental data sample. Inasmuch that $\lambda_{3g} \approx \lambda_{qqg}$, an assumption supported by the analysis results reported in [26], the two independent measurements of r_{3g}/r_{qqg} are consistent within one standard deviation with the expectation of the hadron source approach.

4 Summary

The dependence of the Bose-Einstein correlation of two identical bosons, produced in e^+e^- annihilations, on the configuration of the hadron sources has been taken over from the known cumulants dependence on these sources. Assuming that correlation exists only for hadrons emerging from the same source, leads to a BEC dilution whenever the hadron-pair emerges from two different sources. A relation between the amount of dilution and the BEC extracted dimension r is formulated. Specifically, we identify in the e^+e^- annihilation the outgoing particle jets as the hadron sources which up to centre-of-mass energy of $\sqrt{s} = 91$ GeV are essentially limited to three.

The comparison of the hadron source approach to the experimental findings is somewhat hindered by the need to know the genuine values of the chaoticity λ parameters, and in some of the cases also the distribution of the true multiplicity, both as a function of the jet energy. In addition, the extension of the hadron source approach to e^+e^- annihilations at energies above 91 GeV seems to require the inclusion of configurations higher than three hadron jet sources which are not dealt with here.

Insofar that λ remains the same over the whole energy range from 10 to 91 GeV, the hadron source approach expects the BEC extracted r value to be only slightly dependent on energy in accordance with the experimental observations. As for the hadronic Z^0 decay, the seen rise of r by some 10% when shifting from the two to the three hadron-jet configuration, is also expected from the hadron source approach. Finally the model is consistent within errors with the experimental findings for the ratio between the r values obtained for the three gluons and to the one photon hadronic decay modes of the $\Upsilon(9.46)$.

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